常 用 數 學 公 式

$a)\quad \ln x = \int_1^x \frac{dt}{t} \qquad (\underline{x>0})$

$\quad special\ cases:\ \ln(1)=0,\quad \ln(0)=-\infty,\quad \ln(\infty)=+\infty$

$b)\quad \ln(xy)=\ln x+\ln y \qquad \ln(x^r)=r\cdot\ln x$

$\quad \ln(\frac{x}{y})=\ln x-\ln y$

$a)\quad e^1\approx 2.718281828\ ,\quad e^0=1\ ,\quad e^{j\theta}=\cos\theta+j\sin\theta$

$b)\quad e^x=\exp(x)$

$c)\quad e^{\ln y}=y \quad\Leftrightarrow\quad \exp(\ln y)=y$

$\quad \ln e^y=y \quad\Leftrightarrow\quad \ln(\exp(y))=y$

$\sin(A+B)=\sin A\cdot\cos B+\cos A\cdot\sin B$

$\sin(A-B)=\sin A\cdot\cos B-\cos A\cdot\sin B$

$\cos(A+B)=\cos A\cdot\cos B-\sin A\cdot\sin B$

$\cos(A-B)=\cos A\cdot\cos B+\sin A\cdot\sin B$

$\sin A\cdot\cos B=\frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$

$\cos A\cdot\sin B=\frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$

$\cos A\cdot\cos B=\frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$

$\sin A\cdot\sin B=-\frac{1}{2}\left[\cos(A+B)-\cos(A-B)\right]$

$assume\quad \begin{cases} x=A+B \\ y=A-B \end{cases} \quad then \quad \begin{cases} A=(x+y)/2 \\ B=(x-y)/2 \end{cases}$

$\sin x+\sin y=2\cdot\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2}$

$\sin x-\sin y=2\cdot\cos\frac{x+y}{2}\cdot\sin\frac{x-y}{2}$

$\cos x+\cos y=2\cdot\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}$

$\cos x-\cos y=-2\cdot\sin\frac{x+y}{2}\cdot\sin\frac{x-y}{2}$

$\sin 2\theta = 2\sin\theta\ \cos\theta$

$\cos 2\theta = \cos^2\theta-\sin^2\theta = 2\cos^2\theta-1 = 1-2\sin^2\theta$

$\cos^2\theta=\frac{1+\cos 2\theta}{2}\ ,\quad \sin^2\theta=\frac{1-\cos 2\theta}{2} \quad\Leftrightarrow\quad \underline{\underline{\sin^2\theta+\cos^2\theta=1}}\ \cdots(a)$

$\left((a)\div\sin^2\theta\right)\Rightarrow \underline{\underline{1+\tan^2\theta=\sec^2\theta}}\ ,\quad \left((a)\div\cos^2\theta\right)\Rightarrow \underline{\underline{1+\cot^2\theta=\csc^2\theta}}$

常 用 微 分 公 式	
$d(fg) = g\,df + f\,dg$ $du_1 + du_2 = d(u_1 + u_2)$ $\frac{dC}{dx} = 0 \Leftrightarrow dC = 0 \quad (C: constant)$ $d(x+C) = dx \Leftrightarrow dx = d(x+C)$	$\frac{de^x}{dx} = e^x \Leftrightarrow de^x = e^x dx$ $\frac{d\ln x}{dx} = \frac{1}{x} \Leftrightarrow \frac{1}{u}du = d\ln u$
$\frac{dx^n}{dx} = nx^{n-1} \Leftrightarrow dx^n = nx^{n-1}dx$ $\frac{dx^2}{dx} = 2x \Leftrightarrow dx^2 = 2x\,dx$ $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2} \Leftrightarrow d\left(\frac{1}{x}\right) = \frac{-1}{x^2}dx$	$d(xy) = y\,dx + x\,dy$ $d\left(x^m y^n\right) = m\cdot x^{m-1}y^n dx + n\cdot y^{n-1}x^m dy$ $\therefore m\cdot y dx + n\cdot x dy = \frac{d(x^m y^n)}{x^{m-1}y^{n-1}}$ $d\left(\frac{y}{x}\right) = \frac{x\,dy - y\,dx}{x^2}$ $d\left(\frac{x}{y}\right) = \frac{y\,dx - x\,dy}{y^2}$
$\frac{d\sin x}{dx} = \cos x \Leftrightarrow d\sin x = \cos x\,dx$ $\frac{d\cos x}{dx} = -\sin x \Leftrightarrow d\cos x = -\sin x\,dx$ $\frac{d\tan x}{dx} = \sec^2 x \Leftrightarrow d\tan x = \sec^2 x\,dx$ $\frac{d\cot x}{dx} = -\csc^2 x \Leftrightarrow d\cot x = -\csc^2 x\,dx$ $\frac{d\sec x}{dx} = \sec x\tan x \Leftrightarrow d\sec x = \sec x\tan x\,dx$ $\frac{d\csc x}{dx} = -\csc x\cot x \Leftrightarrow d\csc x = -\csc x\cot x\,dx$	$\frac{d\sin^{-1}x}{dx} = \frac{1}{\sqrt{1-x^2}} \Leftrightarrow \frac{dx}{\sqrt{1-x^2}} = d\sin^{-1}x$ $\frac{d\tan^{-1}x}{dx} = \frac{1}{1+x^2} \Leftrightarrow \frac{dx}{1+x^2} = d\tan^{-1}x$ $\frac{d\sec^{-1}x}{dx} = \frac{1}{x\sqrt{x^2-1}} \Leftrightarrow \frac{dx}{x\sqrt{x^2-1}} = d\sec^{-1}x$ $\frac{d\cos^{-1}x}{dx} = \frac{-1}{\sqrt{1-x^2}} \Leftrightarrow \frac{-dx}{\sqrt{1-x^2}} = d\cos^{-1}x$ $\frac{d\cot^{-1}x}{dx} = \frac{-1}{1+x^2} \Leftrightarrow \frac{-dx}{1+x^2} = d\cot^{-1}x$ $\frac{d\csc^{-1}x}{dx} = \frac{-1}{x\sqrt{x^2-1}} \Leftrightarrow \frac{-dx}{x\sqrt{x^2-1}} = d\csc^{-1}x$

微積分定理與公式
$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$ $\frac{d}{dx}[f \cdot g] = g \cdot \frac{d}{dx}f + f \cdot \frac{d}{dx}g \quad \Leftrightarrow \quad [f \cdot g]' = f' \cdot g + f \cdot g'$ $\left[\frac{f}{g}\right]' = \frac{f' \cdot g - f \cdot g'}{g^2} \quad \Rightarrow g(x) \neq 0$ chain rule : if $y = y(u)$ and $u = u(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
$\frac{d\,F(x)}{dx} = f(x) \quad \Rightarrow \quad \int f(x)\,dx = F(x) + C$ $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$, a : constant. $\frac{d}{dx}\int f(x)\,dx = f(x) \quad => \quad d\int f(x)\,dx = f(x)\,dx$ $f(x)\,dx = d\left[\int f(x)\,dx\right]$ $\int \frac{d\,f(x)}{dx}dx = f(x) + C \quad => \quad \int d\,f(x) = f(x) + C$ (C : integral constant) $\begin{cases} y = y(x) & dy = y' \cdot dx \\ u = u(x,y) & du = \frac{\partial u}{\partial x} \cdot dx + \frac{\partial u}{\partial y} \cdot dy \end{cases}$ $\int u \cdot dv = u \cdot v - \int v \cdot du$ (integral by parts) $\frac{d}{dx}\int_{p(x)}^{b(x)} f(x,t)dt = f\left(x, b(x)\right)\frac{db}{dx} - f\left(x, p(x)\right)\frac{dp}{dx} + \int_{p(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)dt$
$\frac{dC}{dx} = 0$ $\frac{d\,e^x}{dx} = e^x \xrightarrow{\text{chain rule}} \frac{d\,e^{ax}}{dx} = a\,e^{ax}$ $\frac{d\,x^n}{dx} = n\,x^{n-1}$ $\frac{d\,a^x}{dx} = \ln a \cdot a^x$

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$\frac{d\sin x}{dx}=\cos x \xrightarrow{\text{chain rule}} \frac{d\sin\omega x}{dx}=\omega\cdot\cos\omega x$
$\frac{d\cos x}{dx}=-\sin x \xrightarrow{\text{chain rule}} \frac{d\cos\omega x}{dx}=-\omega\cdot\sin\omega x$
$\frac{d\tan x}{dx}=\sec^2 x \xrightarrow{\text{chain rule}} \frac{d\tan\omega x}{dx}=\omega\cdot\sec^2\omega x$
$\frac{d\cot x}{dx}=-\csc^2 x \xrightarrow{\text{chain rule}} \frac{d\cot\omega x}{dx}=-\omega\cdot\csc^2\omega x$
$\frac{d\sec x}{dx}=\sec x\cdot\tan x \xrightarrow{\text{chain rule}} \frac{d\sec\omega x}{dx}=\omega\cdot\sec\omega x\cdot\tan\omega x$
$\frac{d\csc x}{dx}=-\csc x\cdot\cot x \xrightarrow{\text{chain rule}} \frac{d\csc\omega x}{dx}=-\omega\cdot\csc\omega x\cdot\cot\omega x$
$\frac{d\sin^{-1}x}{dx}=\frac{1}{\sqrt{1-x^2}} \xrightarrow{\text{chain rule}} \frac{d\sin^{-1}\omega x}{dx}=\frac{\omega}{\sqrt{1-(\omega x)^2}}$
$\frac{d\cos^{-1}x}{dx}=\frac{-1}{\sqrt{1-x^2}} \xrightarrow{\text{chain rule}} \frac{d\cos^{-1}\omega x}{dx}=\frac{-\omega}{\sqrt{1-(\omega x)^2}}$
$\frac{d\tan^{-1}x}{dx}=\frac{1}{1+x^2} \xrightarrow{\text{chain rule}} \frac{d\tan^{-1}\omega x}{dx}=\frac{\omega}{1+(\omega x)^2}$
$\frac{d\cot^{-1}x}{dx}=\frac{-1}{1+x^2} \xrightarrow{\text{chain rule}} \frac{d\cot^{-1}\omega x}{dx}=\frac{-\omega}{1+(\omega x)^2}$
$\frac{d\sec^{-1}x}{dx}=\frac{1}{x\sqrt{x^2-1}} \xrightarrow{\text{chain rule}} \frac{d\sec^{-1}\omega x}{dx}=\frac{1}{x\sqrt{(\omega x)^2-1}}$
$\frac{d\csc^{-1}x}{dx}=\frac{-1}{x\sqrt{x^2-1}} \xrightarrow{\text{chain rule}} \frac{d\csc^{-1}\omega x}{dx}=\frac{-1}{x\sqrt{(\omega x)^2-1}}$
Binomial formula : $(x+y)^n=\sum_{k=0}^{n}C_k^n x^k y^{n-k}=\sum_{k=0}^{n}C_k^n x^{n-k}y^k$
Leibniz's formula : $\frac{d^n}{dx^n}(f\cdot g)=(f\cdot g)^{(n)}=\sum_{k=0}^{n}C_k^n f^{(k)}g^{(n-k)}$
where $C_k^n=\binom{n}{k}=\frac{n!}{k!(n-k)!}$

* ***Taylor's*** *series expansion* :

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}\cdot(x-a)^n$$

$$=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f^{(3)}(a)}{3!}(x-a)^3+\cdots\cdots$$

where $n!=n(n-1)(n-2)\cdots(2)(1).$

f(x) is an infinitely differentiable function.

some important expansions:

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\cdots$$

$$\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\cdots$$

$$\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots\cdots$$

$$(1+x)^p=1+px+\frac{p(p-1)}{2!}x^2+\frac{p(p-1)(p-2)}{3!}x^3+\cdots\cdots$$

If $\frac{d}{dx}F(x)=f(x)$, then $\int f(x)dx=F(x)+C \quad\Rightarrow\quad \left[\frac{d}{dx}C=0\right]$

$$\int x^n dx=\frac{1}{n+1}x^{n+1}+C \qquad (n\neq -1)$$

$$\int\frac{dx}{x}=\ln|x|+C \quad\Rightarrow\quad \int\frac{du}{u}=\ln|u|+C$$

$$\int e^{ax}dx=\frac{1}{a}\cdot e^{ax}+C$$

$$\int a^x dx=\frac{1}{\ln a}\cdot a^x+C \quad\Rightarrow\quad a^x=e^{\ln(a^x)}=e^{x\cdot\ln a}$$

$$\int\frac{1}{\sqrt{1-x^2}}dx=\sin^{-1}x+C \qquad \int\frac{-1}{\sqrt{1-x^2}}dx=\cos^{-1}x+C$$

$$\int\frac{1}{1+x^2}dx=\tan^{-1}x+C \qquad \int\frac{-1}{1+x^2}dx=\cot^{-1}x+C$$

$$\int\frac{1}{x\sqrt{x^2-1}}dx=\sec^{-1}x+C \qquad \int\frac{-1}{x\sqrt{x^2-1}}dx=\csc^{-1}x+C$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cdot\cos bx + b\cdot\sin bx\right] + C$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cdot\sin bx - b\cdot\cos bx\right] + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \tan x\,dx = \ln\left|\sec x\right| + C \quad \Rightarrow \quad \int \sec^2 x\,dx = \tan x + C$$

$$\int \cot x\,dx = \ln\left|\sin x\right| + C \quad \Rightarrow \quad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x\,dx = \ln\left|\sec x + \tan x\right| + C \quad \Rightarrow \quad \int \sec^2 \omega x\,dx = \frac{1}{\omega}\tan \omega x + C$$

$$\int \csc x\,dx = -\ln\left|\csc x + \cot x\right| + C \quad \Rightarrow \quad \int \csc^2 \omega x\,dx = -\frac{1}{\omega}\cot \omega x + C$$

$$\int \sin \omega x\,dx = -\frac{1}{\omega}\cdot\cos \omega x + C$$

$$\int \cos \omega x\,dx = \frac{1}{\omega}\cdot\sin \omega x + C$$

$$\int \tan \omega x\,dx = \frac{1}{\omega}\ln\left|\sec \omega x\right| + C$$

$$\int \cot \omega x\,dx = \frac{1}{\omega}\ln\left|\sin \omega x\right| + C$$

$$\int \sec \omega x\,dx = \frac{1}{\omega}\ln\left|\sec \omega x + \tan \omega x\right| + C$$

$$\int \csc \omega x\,dx = -\frac{1}{\omega}\ln\left|\csc \omega x + \cot \omega x\right| + C$$